The analog-digital distinction: Can positive procedures be transformed into approximation procedures?[1]

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1. Introduction

According to the 'received view' of the analog-digital distinction, analog schemes of mental representations are *continuous* while digital schemes are (finite and) *discrete*. In other words,, continuous schemes comprise sets of representational types bijective with the real numbers, while discrete schemes comprise sets bijective with finite subsets of the natural numbers.[2]

The received view is grounded in the mathematical foundations of computational theory. In contrast, some recent accounts consider how 'analog' and 'digital' appear in 'implementational contexts', i.e., situations where the terms are being used to describe physical devices such as brains or artifacts (e.g., Maley 2011, Katz 2008). This general strategy can be viewed as an intellectual descendent of Haugeland's influential analysis.[3] For instance, while discussing Goodman's (1968) version of the received view, Haugeland chastises those who

> betray a mathematician's distaste for the nitty-gritty of *practical* devices. But *digital*, like *accurate, economical,* or *heavy-duty*, is a mundane engineering notion, root and branch. It only makes sense as a practical means to cope with the vagaries and vicissitudes, the noise and drift, of earthly existence. The definition should reflect this character. (1981, p. 80; the specific target of this comment is Goodman)

In other words, Haugeland's strategy is to forgo mathematical logic in favor of considering *physical devices*. In today's intellectual climate – where philosophy of mind often walks hand-in-hand with cognitive psychology – it is easy to see how this strategy is attractive: It opens up the possibility that the analog-digital distinction, which is supposed to concern mental representation, can be understood by considering research on physical devices such as brains. So, for instance, Katz (2008) references research on numerical reasoning in support of his account, and Maley (2011) appeals to the literature on mental rotation.

In this brief paper I argue that, despite initial appearances, Haugeland's account is properly viewed as an articulation of the practical consequences of the received view, not an endorsement of an 'implementation-based' alternative to it. Like Goodman's (1968) and Lewis' (1971), Haugeland's

1 This brief paper can be viewed as an appendix to my (considerably longer) prior work on the analog-digital distinction.
2 This is the sense the terms are used in computer science and electrical engineering, i.e., in those domains concerned with implementing analog and digital devices, so the assumption is at least consistent with practice (cf. Argawal & Lang 2005). This version of the received view is, as far as I can tell, closer to Lewis (1971) than Goodman (1971), but discussion of this topic is outside the scope of the present paper.
3 Katz (2008) makes the debt explicit, arguing that his account is implied by Haugeland's. If the interpretation of Haugeland offered here is correct, then Katz is mistaken.

account should motivate analyses that aim to respect the formal foundations of the analog-digital distinction.

2.Analog and digital, approximate and positive

Haugeland writes,

> we can define a *digital device* as: (1) a set of types; (2) a set of feasible procedures for writing and reading tokens of those types, and (3) a specification of suitable operating conditions, such that (4) under those conditions, the procedures for the write-read cycle are positive and reliable. (1981/1998, p. 78; original emphasis, formatting adjusted)

In this device the set of types is the set of representations. It is the mathematically-construed structure of this set that forms the basis for the received view, but, as is clear from this passage, Haugeland is silent about this structure, instead adding a series of engineering restrictions to the definition of a digital device.

Analog devices also begin with a set of types and procedures for writing and reading tokens of those types, but their engineering constraints differ insofar as they use *approximation procedures* rather than the *positive procedures* specified in clause (4) of the digital definition (p. 83). Consequently, for Haugeland, the analog-digital distinction turns on a difference in feasible and reliable write-read procedures, not the abstract structure of the set of symbols. What, then, is the difference between these two kinds of procedure?

Haugeland defines a positive procedure as follows:

> A *positive procedure* is one which can succeed absolutely and without qualification – that is, not merely to a high degree, with astonishing precision, or almost entirely, but *perfectly*, one hundred percent! (p. 77)

In other words, a procedure is positive if it can (reliably) succeed in categorizing a given token as an instance of its intended representational type, without error. In contrast,

> *approximation procedures* [are] ones which can "come close" to perfect success. More specifically, there is some notion of margin of error (degree of deviation from perfect success) such that: (1) the smaller this margin is set, the harder it is to stay within it; (2) available procedures can (reliably) stay within a pretty small margin; (3) there is no limit to how small a margin better (future, more expensive) procedures may be able to stay within; but (4) the margin can never be zero – perfect procedures are impossible. (p. 83, original emphasis, formatting adjusted)

To sum, on Haugeland's account, the difference between analog and digital representations turns on a difference in the procedure physical devices use to type token instances of representations. Analog devices use procedures that allow for uncertainty in typing (i.e., a margin of error), while digital devices do not.

This suggests that whether a representational scheme is analog or digital depends solely on facts about the physical makeup of a device: If the materials and technology available do not allow for reliable typing (or if the design goals do not require reliable typing), then the device is analog, but if they do support reliable typing, then the device is digital. This interpretation gains *prima facie* support from Haugeland's illustration of positive procedures:

> Clearly, whether something is a positive procedure depends on what counts as success. Parking the car in the garage (in the normal manner) is a positive procedure, if getting it all the way in is all it takes to succeed; but if complete success requires getting it exactly centered between the walls, then no parking procedure will be positive. There is no positive procedure for cutting a six-foot board, but there are plenty for cutting boards six feet, plus or minus an inch. The '*can* succeed' means feasibly, and that will depend on the technology and resources available. (p. 77)

In this example, the feasibility of there being a positive procedure varies with the criteria for success. Furthermore, if the criteria are too stringent with respect to what can be accomplished given the physical constitution of the system, then the best one could hope for is approximation within some margin of error. Thus it seems we can transform positive procedures into approximation procedures – and approximation procedures into positive ones – by adjusting how much variation in physical state is tolerated by the device (e.g., Katz 2008, p. 406).

If this interpretation is correct, the division between analog and digital is grounded squarely in the physical devices themselves, and *not* in the idealized mathematical structure of their representational schemes. More generally, if this interpretation is correct, Haugeland's account provides support for the strategy of considering contexts where 'analog' and 'digital' are used in the service of describing actual physical mechanisms, because understanding the nature of the distinction depends on how those devices differ in their information-processing mechanisms.

3. The received view explains the positive-approximate distinction

The interpretation of Haugeland just offered implies that the received view is significantly off-target. If the analog-digital distinction turns on the presence of approximation or positive procedures, then the received view's focus on the formal structure of a representational scheme is orthogonal to the distinction: Any device using a discrete representational scheme with an approximation procedure is analog. Furthermore, unless this observation can be made mathematically rigorous, the general strategy of grounding the analog-digital distinction in mathematical logic fails as well.

However, the notion that the analog-digital distinction turns merely on read-write procedure is counterintuitive given everyday examples of analog and digital devices. For instance, if we set out to build a digital computer only to discover that we cannot reliably type voltages as high (1) or low (0), we intuitively have a malfunctioning digital computer, not an analog computer. Similarly, if we take a slide rule (a prototypical analog computer) and increase its margin of error (e.g., by using components that are not as precisely machined), the device does not become digital; rather, the result is simply a less precise analog device.

The received view has the benefit of accommodating these intuitions. Suppose a digital system is indeed one that uses a finite and discrete representational scheme. Then, when we approach the problem of physically implementing such a scheme, it is imperative that the device reliably identify token states as instances of one or the other of these representational types – there is no wiggle room. If the device cannot do this, then it is *failing* in the engineering goal of realizing a discrete representational scheme.

Similarly, suppose the intention is to realize a device that uses a continuous representational scheme. Since representations differ by arbitrarily small degrees, any variation in the physical states used to implement these representations is a variation in representational type, and given the fact that the physical world is a noisy place, there will *always* be such fluctuations. Consequently, the engineering goal of reliably typing continuous representations will be impossible to satisfy in practice: The best we can hope for in a physical implementation is minimizing the margin of error intrinsic to the device, e.g., through the development of new technology or error models.

The relation to Haugeland's account is clear. According to the received view, if the goal is to realize a digital system, then the engineering problem is not to deliver an *approximation* of the desired symbol type, but rather the *actual* symbol type, i.e., to implement procedures that "can succeed absolutely and without qualification" (Haugeland 1981/1998, p. 77). Given the goal of realizing a discrete set of representations, a possible implementation of a read-write cycle either satisfies this goal of reliably typing token representations or it does not; if it does, the procedure is positive, but if it does not (and it exhibits errors in typing tokens), it is a *failed implementation of a positive procedure, not a successful implementation of an approximation procedure*. In this case, fixing the problem may require adjusting the typing procedures so that they tolerate greater variation in the physical states being typed – but under normal operating conditions the margin of error in typing is *always* zero.

Likewise, if the goal is to realize an analog system, then the engineering problem is not to deliver the actual symbol type but rather an *approximation* of the infinitely precise ideal identified in our formal characterization of the representational scheme. The reason is that every difference in medium state constitutes a difference in instantiated representational type (recall, the types differ by arbitrarily small degrees). Consequently, the margin of error is *always* non-zero, and delivering the actual representational type is physically impossible. As in the case of positive procedures, it is not possible to turn an approximation procedure into a positive one merely by modifying the acceptable margin of error. Rather, *increasing the margin of error only makes the device less precise*.

The passages quoted above are consistent with (and support) this interpretation. For instance, Haugeland claims that, despite there being no limit to how small the margin of error for an approximation procedure may become (since future engineering advances may refine it), it remains the case that "the margin can never be zero – perfect procedures are impossible." (p. 83). This restriction would not make sense if it were possible to transform approximation procedures from one type to the other. It does, however, make sense from the perspective of the received view: The margin can never be zero because the set of representational types to be realized does not allow it; any variation, no matter how small, counts as a variation in type, so every write-read procedure will necessarily involve a margin of error.

Second, the received view explains why Haugeland claims that "approximation procedures are, in a clear sense, the antithesis of positive procedures; the two are exclusive, but of course not exhaustive." (p. 84) They are exclusive because, as a consequence of adopting sets of representations with certain mathematical properties, no digital device will ever involve the use of approximation procedures, and no analog device will ever involve the use of positive procedures – the procedures constitute mutually exclusive design goals. They are not exhaustive because, for example, a failed attempt at physically realizing a positive procedure is neither a positive nor an approximation procedure.

Finally, Haugeland's examples of positive procedures – sawing wood and parking cars – are just that: Examples of how the successful implementation of positive procedures requires taking into account the available technology. The message is not that positive procedures can be transformed into approximation procedures by making the typing procedure more strict (and hence making the results of that procedure approximate); rather, the point is simply that, if we want to implement a positive procedure (because we want to implement a digital device), the success criteria for typing tokens must take into account what the available technology can support.

4. Conclusion

Recent work on the analog-digital distinction has adopted a strategy of looking at ‘implementational contexts’, i.e., situations where the terms are used to describe physical devices such as brains or artifacts (Maley 2011, Katz 2008). The intellectual predecessor of this strategy is Haugeland (1981), who, as noted in the introduction, laments the fact that mathematical approaches neglect “the vagaries and vicissitudes, the noise and drift, of earthly existence.” (p. 80) The notion that the analog-digital distinction turns on facts about how a physical device is implemented (rather than its relation to some prior formal distinction) gains *prima facie* support from Haugeland’s account, which focuses on a distinction between ways physical devices type token symbols. Analog devices use approximation procedures, which type only within a margin of error, while digital devices use positive procedures, which reliably type without error.

Thus stated, it seems that analog and digital devices are interchangeable in the sense that one can be transformed into the other by changing the acceptable margin of error. However, this interpretation runs into difficulties when judged by the tribunal of common sense. First, adjusting a digital device so that it no longer reliably types leads to a malfunctioning digital device, not an analog device; second, increasing the margin of error in an analog device leads to a less precise analog device, not a digital one.

In contrast, the received view accommodates these intuitions and illuminates Haugeland’s distinction between approximate and positive procedures. Analog representations are ideally (i.e., formally) continuous, while digital representations are ideally discrete. The choice of whether to implement an analog or a digital system thus leads to distinct engineering considerations, namely, differences in how to cope with the noise intrinsic to the physical world. Since analog representations vary in the presence of noise, the engineering goal is to accommodate this noise by minimizing the margin of error, i.e., by realizing useful approximation procedures to the extent the available technology allows. Since digital representations require reliable typing prior to their semantic interpretation, the engineering goal is reliably type representational states, i.e., to realize positive procedures. Because each of these engineering goals are *goals*, they can be satisfied or not, which explains why increasing the margin of error on an analog device leads to a less precise analog device, and why allowing for typing errors in a digital device results in a malfunctioning digital device. In short, Haugeland’s account spells out the consequences of the received view for engineering.

To sum, we should be suspect of any approach to the analog-digital distinction that eschews its formal foundations, since those foundations articulate the principles guiding the implementation of analog and digital devices.

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